JOURNAL OF INTERNATIONAL ACADEMIC RESEARCH FOR MULTIDISCIPLINARY
Impact Factor 2.417, ISSN: 2320-5083, Volume 3, Issue 12, January 2016

WAVELET TRANSFORM AND ITS APPLICATIONS IN VARIOUS FIELDS

AMANDEEP KAUR*

*Dept. of Mathematics, Guru Nanak Dev University, Amritsar, India

ABSTRACT

Being a powerful mathematical tool Wavelet Transform has fascinated the scientific, engineering and mathematics community with its versatile applicability. Application areas for wavelets have been growing for the last ten years at a very rapid rate. They have got high attention in various fields such as numerical analysis, signal processing, image processing, physics, engineering, economics and finance, statistics, differential equations, neural networks, sampling theory etc. This paper describes a brief introduction to wavelet transform and illustrates the applications of wavelets transform in some fields.

KEYWORDS: Fourier Transform, Short-Time Fourier Transform, Continuous Wavelet Transform, Discrete Wavelet Transform, WNN.

1. HISTORICAL INTRODUCTION

Wavelet-based analysis of signals is an interesting and relatively new mathematical tool. This new concept can be viewed as a synthesis of various ideas which originated from different fields including mathematics, physics and engineering. In many applications, especially in the time-frequency analysis of a signal, the standard Fourier transform is not adequate because it allows analysis of signals under the main assumption that the observed signal is stationary over the time period of the analysis. This assumption is not valid for many practical signals as most of the real world signals are non stationary. Also Fourier transform gives only frequency information of the signal, not regarding time. To overcome these drawbacks, Dennis Gabor in 1946, first introduced a modified time dependent version of it, namely, short-time Fourier transform known [STFT] later as Gabor transform. The short-time Fourier transform uses a fixed window function with respect to frequency and applies the Fourier transform to the windowed signal. Once the window function is chosen both the time as well as frequency resolutions become fixed for all frequencies and times respectively. As a consequence, the short- time Fourier transform does not allow any change in time or frequency resolutions.

Wavelet transform is an alternative approach to the short-time Fourier transform to overcome the resolution problem. It is capable of providing the time and frequency information simultaneously, hence giving time-frequency representation of the signal. Moreover, it does not require the stationarity of the signal. In contrast to the fixed time frequency partition of the short-time Fourier transform, the Wavelet transform analyzes the signal at different resolutions using multiresolution analysis. The multiresolution analysis approach may overcome the resolution problem as it adaptively partitions the time frequency plane, using short windows at high frequencies and long windows at low frequencies and thus letting both time and frequency resolutions to vary in the time –frequency plane.

In 1982, Jean Morlet, a French geophysical engineer, first introduced the idea of wavelet transform as a new mathematical tool for seismic signal analysis. A French Physicist, Alex Grossman, quickly recognized the importance of the Morlet wavelet transform and developed an exact inversion formula for the wavelet transform. The joint work of Morlet and Grossman led to a detailed mathematical study of the continuous wavelet transform and their applications. They proved that like Fourier analysis, the wavelet analysis has provided a new method for decomposing a signal. Yves Meyer, a French Mathematician, widely acknowledged as one of the founders of wavelet theory, recognized the connection between Morlet's wavelets and earlier mathematical wavelet such as those in the work of Little wood and Paley. He discovered a new kind of wavelet, with a mathematical property called orthogonality that made the wavelet transform an easy to work with and manipulate as a Fourier transform. In 1986, Stephane Mallat brought out the relation between wavelet theory and filter bank theory used in image processing applications. Mallat constructed wavelet decomposition and reconstruction algorithms using Multiresolution Analysis (MRA), which is known as the heart of wavelet theory. A few months later, Battle and Lemarie constructed spline orthogonal wavelets with exponential decay. The next major achievement of wavelet analysis was due to Daubechies who discovered a whole new class of wavelets which are non orthogonal and could be implemented using short digital filters. Her work had a tremendous positive impact on the study of wavelets and their applications

The rest of the paper structures as follows: the next section describes the wavelet theory and its properties. The third section discusses the applications of wavelet transform in diverse fields. The last section concludes the paper.

2. Wavelet Theory and its properties

Wavelets mean 'small waves'. They have finite length and oscillatory behaviour. So wavelet analysis is about analyzing the signal with short duration finite energy functions.

They transform the signal into another more useful form. This transformation of the signal is called wavelet transform. Wavelets can be manipulated in two ways. One is translation, means shifting along the time axis and other is scaling, changing in scale. Like Fourier transform, wavelet transform deals with expansions of functions in terms of set of basis functions. Unlike Fourier transform, wavelet transform expands not in term of sinusoidal but in term of wavelets, which are generated in the form of translation and dilation of a fixed function called the mother wavelet. The wavelets obtained in this way have special scaling properties. They are localized in time and frequency, permitting a closer connection between the function (signal) being represented and their coefficients.

The wavelet transform utilizes a basic function, $\psi \in L_2(\mathbb{R})$, called wavelet or mother wavelet that is stretched and shifted to capture features that are local in time and local in frequency. This function satisfies the following properties:

(i) $$C_\psi = \int_0^\infty \frac{|\hat{\psi}(\omega)|}{\omega} d\omega < \infty$$

where $\hat{\psi}$ is the Fourier transform of ψ. This condition, called admissibility condition, ensures that $\hat{\psi}(\omega)$ goes to zero quickly as $\omega \to 0$. In fact, to guarantee that $C_\psi < \infty$, it is necessary $\hat{\psi}(0) = 0$ which is equivalent to

$$\int_{-\infty}^{\infty} \psi(t) dt = 0$$

(ii) Wavelet function must have unit energy. That is

$$\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1$$

By combining several combinations of shifting and stretching of the mother wavelet, the wavelet transform is able to capture all the information in the time series and associate it with specific time horizons and locations in time. There are two types of wavelet transform-continuous wavelet transform and discrete wavelet transform which are briefly discussed as below:

Continuous Wavelet Transform (CWT)

If $\psi \in L_2(\mathbb{R})$ satisfies the properties discussed above, then the continuous wavelet transform of a real signal $f(t) \in L_2(\mathbb{R})$ is defined as:

$$W(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} \bar{\psi}\left(\frac{t-b}{a}\right) f(t)dt, \qquad a > 0, b \in \mathbb{R} \qquad (2.1)$$

Where $\bar{\psi}$ denotes the complex conjugate of ψ .The parameter b corresponds to the time shift and the parameter a corresponds to the scale of the analyzing wavelet and $1/\sqrt{a}$ is for energy conservation i.e. it ensures that the wavelets at every scale all have the same energy. If we define $\psi_{a,b}(t)$ as:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \qquad (2.2)$$

then equation (2.1) can be written as:

$$W(a,b) = \int_{-\infty}^{\infty} \overline{\psi_{a,b}}(t) f(t)dt$$

When the function $\psi(t)$ satisfies admissibility condition (i) the original signal $f(t)$ can be obtained from the wavelet transform $W(a,b)$ by the following inverse formula:

$$f(t) = \frac{1}{C_\psi}\int_{-\infty}^{\infty}\int_{0}^{\infty} W(a,b)\psi_{a,b}(t)\frac{da\,db}{a^2}$$

Discrete Wavelet Transform (DWT)

CWT as mentioned above is a function of two parameters and, therefore, contains a high amount of extra information when analyzing a function. Instead of continuously varying parameters, the signal can be analyzed with a small number of scales with number of translations at each scale. This is discrete wavelet transform (DWT). In CWT there are wavelet coefficients for every (a,b) combination whereas in DWT, there are wavelet coefficients only at very few points. The DWT produces only the minimal number of coefficients necessary to reconstruct the original signal. It may be viewed as discretization of CWT parameters via $a = 2^{-j}$ and $b = k2^{-j}$ where j, k are integers. So equation (2.2) becomes

$$\psi_{j,k} = 2^{\frac{j}{2}}\psi(2^j t - k)$$

And corresponding wavelet coefficients of function $f(t)$, denoted by $d_{j,k}$, are:

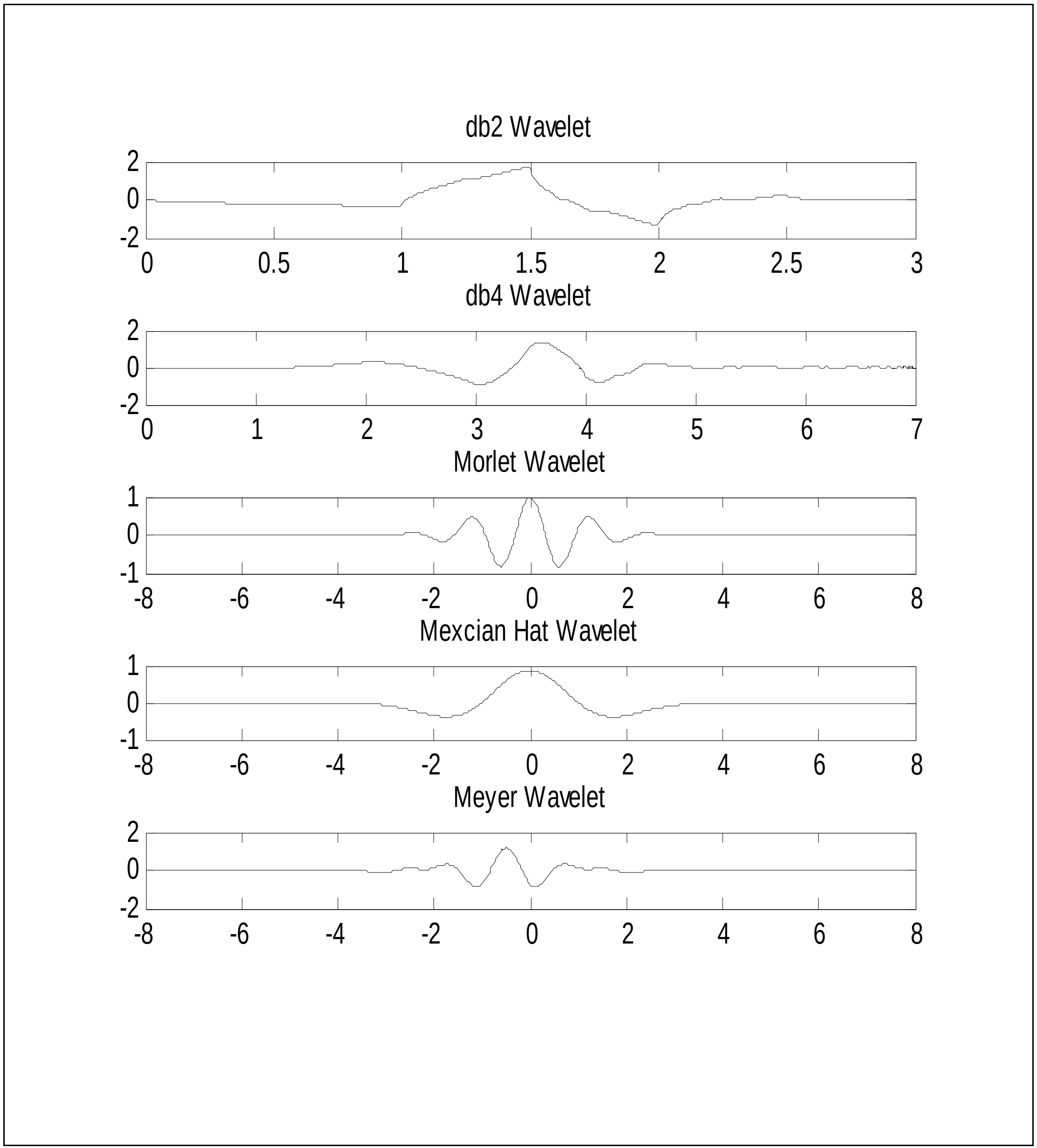
$$d_{j,k} = \int_{-\infty}^{\infty} \psi_{j,k}(t) f(t)\, dt$$

The expression

$$f(t) = \sum_{j\in Z}\sum_{k\in Z} \langle f, \psi_{j,k}(t) \rangle\, \psi_{j,k}(t)$$

is called the wavelet representation of $f(t)$

Figure: 1



We have a variety of wavelets that are used for the signal analysis. Choice of particular wavelet depends on the type of application in hand. Figure 1 shows some wavelets.

Here Daubechies 2 (db2) and 4 (db4) are discrete wavelet and Morlet, Mexican Hat and Meyer wavelets are continuous wavelet.

3. Applications of Wavelet Transform

In this section, some applications of wavelet transform are discussed.

Wavelet Neural Network (WNN):

In recent years, wavelets have become a very active subject in scientific and engineering areas. Wavelet Neural Networks (WNN) that combines the theory of wavelet and neural network into one, have received considerable attention. One main application of wavelet neural networks is that of function estimation. WNN are feedfoword neural networks with one hidden layer, comprised of radial wavelet as activation function and a linear output layer. The output layer of the WNN represents the weighted sum of the hidden layer units, i.e. wavelet basis functions. Wavelet networks provide efficient network construction techniques, faster training times, and multiresolution analysis capabilities.

There are two kind of wavelet neural network structure.

- In the first there are fixed wavelet bases, where the translation and dilation parameters of wavelet basis are fixed and only the output layer weights are adjustable.
- Another type is the variable wavelet bases, where the translation and dilation parameters and the output layer weights are adjustable.

Wavelet neural networks have been mostly used in nonparametric statistical analysis and regression. Since WNN are best at identifying patterns in data, they are well suited for forecasting needs. They also have been successfully applied in robotics, modeling and machine learning. Abiyev R. [1] has used wavelet with neural networks for modelling and prediction of stock prices. K. K. Minu [8] analyses the time series with wavelet neural networks and shows the superiority of this method over other exiting methods. G. Wang [5] proposes a variant of wavelet neural networks, MWFWNN network to select optimal mother wavelet function for solving threat assessment.

Economics and Finance

For economics and finance, the most useful property of wavelet approach is the ability to decompose any series into its time scale components. Mostly economic and financial time series data (financial indices, census data, and spatially distributed econometrics measures) are non stationary, exhibit high complexity and involve both random processes and intermittent deterministic processes. So wavelet methods are most suitable for such type of time series data. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities.

The applications of wavelets in finance are still in its infancy when compared to other subject areas. A good overview of the applications of wavelets in economics and finance is given by J. Ramsey [6]. R. Davidson [10] used the orthogonal dyadic Haar transform to perform semi-nonparametric regression analysis of commodity price behaviour. They showed that wavelet analysis is particularly useful in describing the general features of the commodity prices such as structural breaks, co-movements of prices, and the unstable variance structure. Gencay et al. [10] discussed the use of wavelets in economics and finance with many illustrations and examples. Wavelet-based methods to remove hidden cycles from within financial time series have been developed by Arino [3]. Their methods first decompose the signal into its wavelet coefficients then compute the energy associated with each scale. Shin and Han [14] have investigated exchange rate forecasting using a method which combines wavelet transform, genetic algorithms and artificial neural networks. Some researchers also have used the combination of wavelet with other statistical methods such as ARMA, ARIMA, Exponential smoothing, Trigonometric fit for forecasting the time series data and proved that wavelets do improve the quality of forecasting to a greater extent.

Image compression

Most of the applications of Wavelet Transform is about science and engineering such as audio and image denoising, signal compression, fingerprint compression, edge detection, noise removal, image recognition etc. Wavelet bases are very good at efficiently representing functions that are smooth except for a small set of discontinuities. So any image that has large regions of constant grayscale can be well represented in a wavelet basis. A wavelet basis with sufficient vanishing moments can be used effectively in the transform step in image compression.

It is also possible to find the best wavelet packet basis for an image and use the expansion in that basis as the transform. The advantage of this approach is that the resulting coefficients will be optimized relative to some appropriate measure of efficiency. The goals of image compression are to minimize the storage requirement and communication bandwidth. Compression is achieved by the removal of redundant data. Discrete Wavelet Transform (DWT) is a recently developed compression technique in image compression. DWT image compression includes decomposition (transform of image), Detail coefficients thresholding, and entropy encoding. R. K. Yadav [12] describes image compression using DWT and thresholding techniques. R. A. Devore [13] also describes applications of wavelet transform in compression. Images can be decomposed into four parts by two-dimensional Wavelet Transform. In fact, the decomposition can continue until the size of the sub-image is as small as we want. By setting some parts of its sub-images to zero, we can reduce the quantity of information, In other words we can compress the image by setting the useless data to zero. Edge detection and noise removal are based on the same idea. When we want to detect the edges of the image, we can simply set the diagonal sub-images to zero, and then we can obtain the output image with edges clearly. Also wavelets are widely used in the fields of pattern recognition due to their ability to zoom in finer patterns as well as view the entire global trend

Differential Equations

Wavelet transform, in recent years, has been applied in many areas of applied mathematics including developing numerical schemes to solve the partial differential equations and integral equations. The Galerkin method is one of the best known methods to find the numerical solutions to differential equations. It allows us after the choice of appropriate basis functions to approximate the differential equations as system of linear equations, which we may then solve. Wavelets offer a remedy for different kinds of obstructions encountered in applying Galerkin methods. The matrix that defines this linear system is known as stiffness matrix. In many applications, this matrix can be ill- conditioned. This can be avoided using the Wavelet basis. This makes for a better behaved and numerically easier to solve system. In [2] Siddiqui discusses wavelet approach to solve differential equations. K. Ahmed [7] illustrates the role of wavelets in differential equations and discusses the wavelet procedures with examples.

Medical

Being the non stationary nature of all biological signals, wavelet transform has got a great success in medical sciences. Wavelets have been used for the analysis of electrocardiogram (ECG) for diagnosing cardiovascular disorders, and of electroencephalogram (EEG) for diagnosing neurophysiological disorders. Wavelets have also been used for the detection of microcalcifications in mammograms and processing of computer assisted tomography (CAT), magnetic resonance images (MRI), nuclear magnetic resonance (NMR) and functional images of brain. Moreover, it has found its applications in electromyographic (EMG), clinical sounds, respiratory patterns, blood pressure trends and DNA sequences. P. S. Addison [9] describes all these applications of wavelets.

Statistics

Wavelet is also very popular in the fields of statistics and probability. Many wavelet results have been introduced to the statistical research fields. In statistical function estimation, standard methods like kernel smoothers, orthogonal series methods etc. rely upon certain assumptions about the smoothness of the function estimated. With wavelets such assumptions are relaxed. Wavelets have built in spatial adaptively that allows efficient estimation of functions with discontinuities in derivatives and sharp spikes. The wavelet methods are very useful in non-parametric statistical problems. There are so many statistical models such as density estimation, non-parametric regression estimation, variance estimation, time series model, Gaussian white noise model and diffusion model that were studied in wavelet framework.

Conclusion

Wavelet transform is a mathematical tool that decomposes a signal into a representation which gives more information about the signal. We can use this representation to characterize transient events, reduce noise, compress data, and perform many other operations. The main advantages of wavelet transform methods over the traditional Fourier methods are the use of localized basis functions and the faster computation speed. This paper briefly introduces wavelet transform and discusses its applications in some fields such as neural networks, economics and finance, engineering, medical and statistics.

References

1. A. H. Siddiqi (2004). Applied Functional Analysis: Numerical Methods, Wavelet Methods, and Image processing. Marcel Dekker, Inc., New York.
2. Abiyev H .R. Abiyev H. V (2012). Differential Evaluation Learning of Fuzzy Wavelet Neural Networks for Stock Price Prediction. Journal of Information and Computing Science Vol.7, p. 121-130.
3. Arino M. A, Morettin P and Vidakovic B (1995). Wavelet scalograms and their applications in economic time series. Institute of Statistics and Decisions Sciences.
4. G. Wang, L. Guo and H. Duan(2013). Wavelet Neural Network using Multiple Wavelet Functions in Target Threat Assessment. The Scientific World Journal, Vol. 2013.
5. I. Daubechies (1992). Ten lectures on Wavelets. SIAM, Philadelphia, PA.
6. J. B. Ramsey (2002). Wavelets in economics and finance: past and future. Stud Nonlinear Dynam Economet.
7. K. Ahmed, F. A. Shah (2013). Introduction to Wavelet with Applications. Real World Education publishers.

8. K. K. Minu, M. C. Lineesh and C. Jessy John (2010). Wavelet Neural Networks for Nonlinear Time Series analysis. Applied Mathematical Sciences, Vol. 4, pp- 2485-2495.
9. P.S. Addison (2002). The illustrated Wavelet Transform Handbook. Institute of Physics Publishing Bristol and Philadelphia.
10. R. A. Devore, B. J. Jawerth and B. J. Lucier (1992). Image compression through Wavelet Transform Coding . IEEE Transactions on Information Theory, Vol. 38, No.2.
11. R. Davidson, WC. Labys, and J-B. Lesourd (1998). Wavelet analysis of commodity price behaviour. Comput Econom.
12. R. Gencay, Selcuk, F. and Whitcher, B. (2001). An Introduction to Wavelets and Other Filtering Methods in Finance and Economics. San Diego, CA: Academic Press.
13. R. K. Yadav, S. P. Gangwar and H. V. Singh (2012). Study and analysis of wavelet based image compression techniques. International Journal of Engineering, Science and Technology, Vol.4 No.1 pp 1-7.
14. Shin and Han I.(2000) Optimal signal multiresolution by genetic algorithms to support financial neural networks for exchange- rate forecasting. Expert Syst. Appl. 257-269.