# A NUMERICAL STUDY OF THE EVOLUTION OF THE REAL TIMEINQUANTUM MECHANICSFOR GAUGE THEORY(QUARKSAND GLUONS PLASMA)

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**Abstract**

In this paper, we take the effective Hamiltonian operator until the sixth degree [3] and we apply the perturbation theory (that depends on creation operator $\hat{b}^{+a}_i$ and annihilation operator$\hat{b}^a_i$) on remaining homogenous modes after quantization of the inhomogenous modes and we have concluded the time evolution for the average ensemble of global square operator (the color magnetic energy$\hat{B}^a_i\hat{B}^a_i$) analytically, and we calculated this time evolution with MATLAB software, we found that life quark and gluons plasma of rank $10^{-29}$ sec According to coupling constant values.

**Keywords:** Real times in non-equilibrium, phase transition to quark-gluon-plasma.

## 1-Introduction

Understanding the evolution of real time in problems of non-equilibrium, is very important, that is manifested for example in identification on the process of early universe formation, or in the theoretical study of particles under limited conditions, Aimed to predicting a very short transition of phase of quarks and gluons plasma. These problems are treated within the framework of the groups SU(2) and SU(3) in gauge theories [1-4].

-There are two methods used for processing problems by real time:

1- The first method: depends on Schrodinger representation in quantum mechanics, where the operators are not dependent on time [5,6].

2- The second method: depends on Heisenberg representation in quantum mechanics, where the operators are dependent on time $\hat{A}_H(t)$, and is a processed problems of non- equilibrium, either depending on Green function, or Wigner way [4,7,8].

_The real time evolution of quarks and gluons plasma was studied for the pure gauge theory with the two groups SU(2) and SU(3) in [5-6]. In these studies, the perturbation theory which depends on the creation operator $\hat{D}^+$ and annihilation operator $\hat{D}$, was used .The effective Hamiltonian operator expansion was taken into consideration until the fourth degree.

-In [13] the harmonic oscillator of pure gauge theory with group SU(2) was studied numerically using creation operator $\hat{D}^+$ and annihilation operator $\hat{D}$.

-In [14] We take the effective Hamiltonian operator until the sixth degree with the group SU(2); which represents nine anharmonic oscillators. We used the perturbation theory and two operators of creation and annihilation then calculate and draw the energy levels from $E_{+0}$ to $E_{+11}$ at which this study is still correct and stable. We conclude that the perturbation theory is not broken down when $g \cong 0.549915534$; it still correct until $g = 3.398312073$.

-in this work, the starting point is gauge theory (quarks and gluons plasma) and taking the effective Hamiltonian operator expansion until the sixth degree and we used perturbation theory which depends on creation operator $\hat{D}^{+a}_i$ and annihilation operator $\hat{D}^a_i$.

**2-RESEARCH METHODOLOGY:**

" Introduction to our research in words "

According to [3], the Hamilton operator of gauge theory with group SU(2) can be described in loop $(L^3)$

$$LH_{eff} =$$

$$\frac{1}{2}\left(\frac{1}{g^2(L)}+\alpha_1+n_f f_1\right)^{-1}\Pi_i^a\Pi_i^a + (\alpha_2+n_f f_2)B_i^aB_i^a + \frac{1}{4}\left(\frac{1}{g^2(L)}+\alpha_3+n_f f_3\right)\varepsilon_{ij}(B)\varepsilon_{ij}(B)$$

$$+(\alpha_4+n_f f_4)B_i^aB_i^aB_j^bB_j^b + (\alpha_5+n_f f_5)B_i^aB_j^aB_i^bB_j^b + (\alpha_6+n_f f_6)\sum_i (B_i^aB_i^a)^2$$

$$+(\alpha_7+n_f f_7)\sum_{i=1}(B_i^aB_i^a)^2 B_j^bB_j^b + (\alpha_8+n_f f_8)B_i^aB_i^aB_j^bB_j^bB_k^cB_k^c$$

$$+(\alpha_9+n_f f_9)\varepsilon_{ij}(B)\varepsilon_{ij}(B)B_k^cB_k^c + (\alpha_9+n_f f_9)\varepsilon_{ij}(B)\varepsilon_{ij}(B)B_i^cB_i^c$$

$$+(\alpha_{10}+n_f f_{10})(B_i^aB_i^aB_j^b)^2 + o(B^8) \quad (1)$$

$n_f = 3$ Number of quarks flavors for group SU(2).

where i,j=1,2,3 the guide of local coordinates,.

a,b=1,2,3 are the evidences of group SU(2) generators,.-

$\alpha_1, \dots, \alpha_{10}$ are constants resulted from quantization of inhomogeneous modes of gluons field by the method of paths integration (color magnetic energy).

$f_1, \dots, f_{10}$ are constants resulted from quantization of inhomogeneous modes of quarks field by the method of paths integration (color magnetic energy).

$\alpha_0$ is a constant resulted from quantization of inhomogeneous time derivative modes of gluons field by the method of paths integration (color electric energy).

$f_0$ is a constant resulted from quantization of inhomogeneous time derivative modes of quarks field by the method of paths integration (color electric energy) and it has the following values [3]:

$\alpha_0 = 0.021810428, \alpha_1 = -0.20164661, \alpha_2 = 0.00787143866$
$\alpha_3 = 0.00062862042, \alpha_4 = -0.00073484827, \alpha_5 = 0.0000041676466$
$\alpha_6 = -0.000008817282, \alpha_7 = -0.00012428881, \alpha_8 = -0.00011130266$
$\alpha_9 = -0.00021478176, \alpha_{10} = -0.0012778632$ (2.a)
$f_1 = -0.00006186422, f_2 = 0.042844024, f_3 = -0.0034428844$
$f_3 = 0.00073884288, f_4 = -0.0018838048, f_5 = 0.0000087816812$
$f_6 = -0.000028187826, f_7 = 0.00018884984, f_8 = -0.0000060887872$
$f_9 = -0.000064813046, f_{10} = 0.000064248472$ (2.b)

Note that some values constants $\alpha_m$ ( note that m take values from 0 until 10) is different than it is in the case of pure gauge because of the mutual effect between quarks and gluons.
$F_{ij}^a$ are tensors of the magnetic field intensity represented as [3]:

$$F_{ij}^a = \varepsilon^{abc} B_i^b B_j^c \qquad (3)$$

$B_i^a$ is the operator of homogeneous magnetic field, and $\Pi_i^a$ is the operator of momentum.

$$\varepsilon^{abc} = \begin{cases} 1 \text{ At direct replacement} \\ 0 \text{ When two evidences are equal} \\ -1 \text{ At indirect replacement} \end{cases}$$

$O(B^6)$ indicates that the limits of degree greater than $B^6$ are neglected.
$g^2(L)$ is a coupling constant represented in the following relation [3]:

$$g^{-2}(L) = -2b_0 \ln(L\Lambda_{ms}) + \frac{b_1 \ln[-2\ln(L\Lambda_{ms})]}{2b_0^2} \qquad (4)$$

$$b_0 = \frac{1}{(4\pi)^2}\left(\frac{11}{3}N - \frac{2}{3}n_f\right)$$

$$b_1 = \frac{1}{(4\pi)^4}\left(-\frac{34}{3}N^2 + \frac{10}{3}Nn_f + (N^2-1)n_f/N\right)$$

$\Lambda_{ms} = 74.1705\,\mathrm{MeV}$ represents an identified constant by minimum subtraction of dimension organization.

N=2 Number of dimensions group SU(2), L is the loop length in all spatial directions.

According to this method of Hamilton operator $H_{eff}$, the study of gauge theory with group SU(2) becomes a form of quantum mechanics with group SU(2), This mean that the study of infinite number of particles and freedom degrees (quarks and gluons plasma), has been physically transformed to a study of three global particles. naemly, to confine the study to nine harmonic oscillators. Nine freedom degrees and particularly nine anhamonic oscillators.

, the creation operator can be identified as the following [5,6]:

$$\hat{a}_i^{+} = \sqrt{\frac{\sqrt{\bar{\omega}_i}}{2\hbar}}\,\hat{q}_i - \frac{i}{\sqrt{2\hbar\sqrt{\bar{\omega}_i}}}\,\hat{\Pi}_i \qquad (6)$$

and annihilation operator is defined as:

$$\hat{a}_i = \sqrt{\frac{\sqrt{\bar{\omega}_i}}{2\hbar}}\,\hat{q}_i + \frac{i}{\sqrt{2\hbar\sqrt{\bar{\omega}_i}}}\,\hat{\Pi}_i \qquad (7)$$

$\hbar = 1$ (Plank constant) in a system of natural units .
Where:

$$\bar{\omega}_0 = \left(\frac{1}{g^2(L)} + \alpha_0 + n_f f_0\right)^{-1}, \bar{\omega}_1 = 2(\alpha_1 + n_f f_1)$$

In this case, we find that:

$$\left[\hat{b}_i^a, \hat{b}_j^{+b}\right]_- = \delta_{ij}\delta^{ab}$$
$$\left[\hat{b}_i^a, \hat{b}_j^b\right]_- = \left[\hat{b}_i^{+a}, \hat{b}_j^{+b}\right]_- = 0$$

$\delta_{ij}, \delta^{ab}$ are Kroanker symbols of spatial coordinates and evidences of generating group SU(2). These Kroanker constants are respectively, defined as:

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$
$$\delta^{ab} = \begin{cases} 1 & a = b \\ 0 & a \neq b \end{cases}$$

The result of adding the equation (6) to (7) is the operator of homogeneous magnetic field:

$$\hat{B}_i^a = \sqrt{\frac{\hbar}{2\sqrt{\frac{\alpha_2}{\alpha_1}}}}\left(\hat{b}_i^{+a} + \hat{b}_i^a\right) \qquad (8)$$

The result of subtracting the equation (6) out of (7) is the operator of momentum given as:

$$\hat{\Pi}_i^a = i\sqrt{\frac{\hbar\sqrt{\frac{\alpha_2}{\alpha_1}}}{2}}\left(\hat{b}_i^{+a} - \hat{b}_i^a\right) \qquad (9)$$

**3- RESULTS AND DISCUSSION:**

$\hat{H}_{eff}^{0}$ is the harmonic part of the operator $\hat{H}_{eff}$:-

$$\Delta\hat{H}_{eff}^{0} = \sum_{a=1}^{3}\sum_{i=1}^{3}\left[\frac{1}{2}\left(\frac{1}{g^2(L)} + \alpha_2 + \eta_2 f_2\right)^{-1}\hat{\Pi}_i^a\hat{\Pi}_i^a + \left(\alpha_1 + \eta_1 f_1\right)\hat{B}_i^a\hat{B}_i^a\right]$$

$$\Delta\hat{H}_{eff}^{0} = \sum_{a=1}^{3}\sum_{i=1}^{3}\left[\frac{1}{2}\alpha_2\hat{\Pi}_i^a\hat{\Pi}_i^a + \frac{1}{2}\alpha_1\hat{B}_i^a\hat{B}_i^a\right] \qquad (9)$$

We calculated Hamiltonian operator for harmonic oscillator:

$$\Delta\hat{H}_{eff}^{0} = \hbar\sqrt{\alpha_2\alpha_1}\left(\hat{N} + \frac{9}{2}\right) \qquad (10)$$

Following [5,6], using the equations

$$\hat{b}_i^a\left|\ldots m_i^a \ldots\right\rangle = \sqrt{m_i^a}\left|\ldots m_i^a - 1 \ldots\right\rangle \qquad (11)$$
$$\hat{b}_i^{+a}\left|\ldots m_i^a \ldots\right\rangle = \sqrt{m_i^a + 1}\left|\ldots m_i^a + 1 \ldots\right\rangle \qquad (12)$$
$$\hat{N}_i^a\left|\ldots m_i^a \ldots\right\rangle = m_i^a\left|\ldots m_i^a \ldots\right\rangle \qquad (13)$$
$$\hat{b}_i^a\left|\ldots 0 \ldots\right\rangle = 0 \;, \hat{N}_i^a\left|\ldots 0 \ldots\right\rangle = 0 \qquad (14)$$

- We calculated Hamiltonian operator matrix until sixth degree $H_{n_i^a, m_i^a}$:

$$\Delta H_{n_i^a m_i^a} = \left\langle n_i^a\right|\Delta\hat{H}\left|m_i^a\right\rangle$$
$$= \hbar\sqrt{\alpha_2\alpha_1}\left(\sum_{a=1}^{3}\sum_{i=1}^{3} m_i^a\delta_{n_i^a, m_i^a} + \frac{9}{2}\right) + \left(\alpha_3 + \eta_3 f_3\right)\sum_{a=1}^{3}\sum_{b=1}^{3}\sum_{i=1}^{3}\sum_{j=1}^{3}\left[\frac{\hbar^2}{4\frac{\alpha_1}{\alpha_2}}\right.$$
$$\left(\sqrt{m_i^a+1}\sqrt{m_i^a+2}\sqrt{m_i^a+3}\sqrt{m_i^a+4}\,\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a+4}\right.$$
$$+ m_i^a\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a+2} + (m_i^a+1)\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a+2}$$
$$+ m_i^a(m_i^a-1)\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a} + \sqrt{m_i^a+1}\sqrt{(m_i^a+2)^2}\,\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a+2}$$
$$+ (m_i^a)^2\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a} + m_i^a(m_i^a+1)\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a} + \sqrt{m_i^a}\sqrt{m_i^a-1}(m_i^a-2)\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a-2}$$
$$+ \sqrt{m_i^a+1}\sqrt{m_i^a+2}(m_i^a+3)\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a+2} + m_i^a(m_i^a+1)\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a}$$
$$+ (m_i^a+1)^2\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a} + \sqrt{m_i^a}\sqrt{(m_i^a-1)^2}\,\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a-2}$$
$$+ (m_i^a+1)(m_i^a+2)\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a} + \sqrt{(m_i^a)^2}\sqrt{m_i^a-1}\,\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a-2}$$
$$\left.\left.+ (m_i^a+1)\sqrt{m_i^a}\sqrt{m_i^a-1}\,\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a-2} + \sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a-2}\sqrt{m_i^a-3}\,\delta_{ij}\delta^{ab}\delta_{n_i^a, m_i^a-4}\right)\right]$$
$$+ \left(\alpha_4 + \eta_4 f_4\right)\sum_{a=1}^{3}\sum_{b=1}^{3}\sum_{i=1}^{3}\left[\frac{\hbar^2}{4\frac{\alpha_1}{\alpha_2}}\left(\sqrt{m_i^a+1}\sqrt{m_i^a+2}\sqrt{m_i^a+3}\sqrt{m_i^a+4}\,\delta^{ab}\delta_{n_i^a, m_i^a+4}\right.\right.$$
$$+ m_i^a\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta^{ab}\delta_{n_i^a, m_i^a+2} + (m_i^a+1)\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta^{ab}\delta_{n_i^a, m_i^a+2}$$
$$+ m_i^a(m_i^a-1)\delta^{ab}\delta_{n_i^a, m_i^a} + \sqrt{m_i^a+1}\sqrt{(m_i^a+2)^2}\,\delta^{ab}\delta_{n_i^a, m_i^a-2}$$
$$+ (m_i^a)^2\delta^{ab}\delta_{n_i^a, m_i^a} + m_i^a(m_i^a+1)\delta^{ab}\delta_{n_i^a, m_i^a} + \sqrt{m_i^a}\sqrt{m_i^a-1}(m_i^a-2)\delta^{ab}\delta_{n_i^a, m_i^a-2}$$

$$+\sqrt{m_j^i+1}\sqrt{m_j^i+2}(m_j^i+3)\partial^{2k}C_{n_j^i,m_j^i+2}+m_j^i(m_j^i+1)\partial^{2k}C_{n_j^i,m_j^i}$$
$$+(m_j^i+1)^2\partial^{2k}C_{n_j^i,m_j^i}+\sqrt{m_j^i}\sqrt{(m_j^i-1)^2}\partial^{2k}C_{n_j^i,m_j^i-2}$$
$$+(m_j^i+1)(m_j^i+2)\partial^{2k}C_{n_j^i,m_j^i}+\sqrt{(m_j^i)^2}\sqrt{m_j^i-1}\partial^{2k}C_{n_j^i,m_j^i-2}$$
$$+(m_j^i+1)\sqrt{m_j^i}\sqrt{m_j^i-1}\partial^{2k}C_{n_j^i,m_j^i-2}+\sqrt{m_j^i}\sqrt{m_j^i-1}\sqrt{m_j^i-2}\sqrt{m_j^i-3}\partial^{2k}C_{n_j^i,m_j^i-4}\Big)\Big]$$
$$+(\alpha_s+\eta_s f_s)\Big]$$
$$\sum_{[illegible]}\sum_{[illegible]}\Bigg[\frac{[illegible]}{[illegible]}\Big(\sqrt{m_j^i+1}\sqrt{m_j^i+2}\sqrt{m_j^i+3}\sqrt{m_j^i+4}\sqrt{m_j^i+5}\sqrt{m_j^i+6}C_{n_j^i,m_j^i+6}$$
$$+m_j^i\sqrt{m_j^i+1}\sqrt{m_j^i+2}\sqrt{m_j^i+3}\sqrt{m_j^i+4}C_{n_j^i,m_j^i+4}$$
$$+\sqrt{(m_j^i+1)^2}\sqrt{m_j^i+2}\sqrt{m_j^i+3}\sqrt{m_j^i+4}C_{n_j^i,m_j^i+4}$$
$$+m_j^i(m_j^i-2)\sqrt{m_j^i+1}\sqrt{m_j^i+2}C_{n_j^i,m_j^i+2}$$
$$+\sqrt{m_j^i+1}\sqrt{(m_j^i+2)^2}\sqrt{m_j^i+3}\sqrt{m_j^i+4}C_{n_j^i,m_j^i+4}$$
$$+(m_j^i)^2\sqrt{m_j^i+1}\sqrt{m_j^i+2}C_{n_j^i,m_j^i+2}+m_j^i\sqrt{(m_j^i+1)^2}\sqrt{m_j^i+2}C_{n_j^i,m_j^i+2}$$
$$+m_j^i(m_j^i-1)(m_j^i-2)C_{n_j^i,m_j^i}+\sqrt{m_j^i+1}\sqrt{m_j^i+2}\sqrt{(m_j^i+3)^2}\sqrt{m_j^i+4}C_{n_j^i,m_j^i+4}$$
$$+m_j^i\sqrt{(m_j^i+1)^2}\sqrt{m_j^i+2}C_{n_j^i,m_j^i+2}+\sqrt{(m_j^i+1)^2}\sqrt{m_j^i+2}C_{n_j^i,m_j^i+2}$$
$$+m_j^i(m_j^i-1)^2C_{n_j^i,m_j^i}+\sqrt{(m_j^i+1)^2}\sqrt{(m_j^i+2)^2}C_{n_j^i,m_j^i+2}$$
$$+(m_j^i)^2(m_j^i-1)C_{n_j^i,m_j^i}+m_j^i(m_j^i+1)(m_j^i-1)C_{n_j^i,m_j^i}$$
$$+\sqrt{m_j^i}\sqrt{m_j^i-1}(m_j^i-2)(m_j^i-3)C_{n_j^i,m_j^i-2}$$
$$+\sqrt{m_j^i+1}\sqrt{m_j^i+2}\sqrt{m_j^i+3}\sqrt{(m_j^i+4)^2}C_{n_j^i,m_j^i+4}$$
$$+m_j^i\sqrt{m_j^i+1}\sqrt{(m_j^i+2)^2}C_{n_j^i,m_j^i+2}+\sqrt{(m_j^i+1)^2}\sqrt{(m_j^i+2)^2}C_{n_j^i,m_j^i+2}$$
$$+(m_j^i)^2(m_j^i-1)C_{n_j^i,m_j^i}+\sqrt{(m_j^i+1)}\sqrt{(m_j^i+2)^2}C_{n_j^i,m_j^i+2}+(m_j^i)^3C_{n_j^i,m_j^i}$$
$$+(m_j^i)^2(m_j^i+1)C_{n_j^i,m_j^i}+\sqrt{m_j^i}\sqrt{m_j^i-1}(m_j^i-2)^2C_{n_j^i,m_j^i-2}$$
$$+\sqrt{m_j^i+1}\sqrt{(m_j^i+2)^2(m_j^i+3)}C_{n_j^i,m_j^i+2}+(m_j^i)^2(m_j^i+1)C_{n_j^i,m_j^i}$$
$$+m_j^i(m_j^i+1)^2C_{n_j^i,m_j^i}+\sqrt{m_j^i}\sqrt{(m_j^i-1)^2(m_j^i-2)}C_{n_j^i,m_j^i-2}$$
$$+m_j^i(m_j^i+1)(m_j^i+2)C_{n_j^i,m_j^i}+\sqrt{(m_j^i)^2}\sqrt{m_j^i-1}(m_j^i-2)C_{n_j^i,m_j^i-2}$$
$$+(m_j^i+1)\sqrt{m_j^i}\sqrt{m_j^i-1}(m_j^i-2)C_{n_j^i,m_j^i-2}+\sqrt{m_j^i}\sqrt{m_j^i-1}\sqrt{m_j^i-2}\sqrt{m_j^i-3}(m_j^i-4)C_{n_j^i,m_j^i-4}$$
$$+\sqrt{m_j^i+1}\sqrt{m_j^i+2}\sqrt{m_j^i+3}\sqrt{m_j^i+4}(m_j^i+5)C_{n_j^i,m_j^i+4}$$
$$+m_j^i\sqrt{m_j^i+1}\sqrt{m_j^i+2}(m_j^i+3)C_{n_j^i,m_j^i+2}+\sqrt{(m_j^i+1)^2}\sqrt{m_j^i+2}(m_j^i+3)C_{n_j^i,m_j^i+2}$$
$$+m_j^i(m_j^i-1)(m_j^i+1)C_{n_j^i,m_j^i}+\sqrt{m_j^i+1}\sqrt{(m_j^i+2)^2(m_j^i+3)}C_{n_j^i,m_j^i+2}$$
$$+(m_j^i)^2(m_j^i+1)C_{n_j^i,m_j^i}+m_j^i(m_j^i+1)^2C_{n_j^i,m_j^i}+\sqrt{m_j^i}\sqrt{(m_j^i-1)^2(m_j^i-2)}C_{n_j^i,m_j^i-2}$$
$$+\sqrt{m_j^i+1}\sqrt{m_j^i+2}(m_j^i+3)^2C_{n_j^i,m_j^i+2}+m_j^i(m_j^i+1)^2C_{n_j^i,m_j^i}+(m_j^i+1)^3C_{n_j^i,m_j^i}$$
$$+\sqrt{m_j^i}\sqrt{(m_j^i-1)^3}C_{n_j^i,m_j^i-2}+(m_j^i+1)^2(m_j^i+2)C_{n_j^i,m_j^i}+\sqrt{(m_j^i)^2}\sqrt{(m_j^i-1)^3}C_{n_j^i,m_j^i-2}$$
$$+(m_j^i+1)\sqrt{m_j^i}\sqrt{(m_j^i-1)^3}C_{n_j^i,m_j^i-2}+\sqrt{m_j^i}\sqrt{m_j^i-1}\sqrt{m_j^i-2}\sqrt{(m_j^i-3)^3}C_{n_j^i,m_j^i-4}$$
$$+\sqrt{m_j^i+1}\sqrt{m_j^i+2}(m_j^i+3)(m_j^i+4)C_{n_j^i,m_j^i+2}+m_j^i(m_j^i+1)(m_j^i+2)C_{n_j^i,m_j^i}$$
$$+(m_j^i+1)^2(m_j^i+2)C_{n_j^i,m_j^i}+\sqrt{(m_j^i)^2}\sqrt{(m_j^i-1)^3}C_{n_j^i,m_j^i-2}$$
$$+(m_j^i+1)(m_j^i+2)^2C_{n_j^i,m_j^i}+\sqrt{(m_j^i)^2}\sqrt{m_j^i-1}C_{n_j^i,m_j^i-2}$$
$$+\sqrt{(m_j^i)^2}\sqrt{m_j^i-1}(m_j^i+1)C_{n_j^i,m_j^i-2}+\sqrt{m_j^i}\sqrt{m_j^i-1}\sqrt{(m_j^i-2)^2}\sqrt{m_j^i-3}C_{n_j^i,m_j^i-4}$$
$$+(m_j^i+1)(m_j^i+2)(m_j^i+3)C_{n_j^i,m_j^i}+\sqrt{(m_j^i)^2}\sqrt{m_j^i-1}(m_j^i+1)C_{n_j^i,m_j^i-2}$$

$$+(m_j^x+1)^2\sqrt{m_j^x}\sqrt{m_j^x-1}\hat{a}_{n_j^x,m_j^x-2}+\sqrt{m_j^x}\sqrt{(m_j^x-1)^2}\sqrt{m_j^x-2}\sqrt{m_j^x-3}\hat{a}_{n_j^x,m_j^x-4}$$

$$+\sqrt{m_j^x}\sqrt{m_j^x-1}(m_j^x+1)(m_j^x+2)\hat{a}_{n_j^x,m_j^x-2}$$

$$+\sqrt{(m_j^x)^2}\sqrt{m_j^x-1}\sqrt{m_j^x-2}\sqrt{m_j^x-3}\hat{a}_{n_j^x,m_j^x-4}$$

$$+(m_j^x+1)\sqrt{m_j^x}\sqrt{m_j^x-1}\sqrt{m_j^x-2}\sqrt{m_j^x-3}\hat{a}_{n_j^x,m_j^x-4}$$

$$+\sqrt{m_j^x}\sqrt{m_j^x-1}\sqrt{m_j^x-2}\sqrt{m_j^x-3}\sqrt{m_j^x-4}\sqrt{m_j^x-5}\hat{a}_{n_j^x,m_j^x-6}\Big)\Big]$$

$$+\left(a_r+\eta_r f_r\right)\sum_{k=1}^{3}\left|\frac{\hbar^2}{8\left(\frac{\mu_k}{\omega_r}\right)^{2}}\right.$$

$$\Big(\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y+1}\sqrt{m_k^y+2}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y+2}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y+1}\sqrt{m_k^y+2}m_k^y\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}m_k^y\sqrt{m_k^y+1}\sqrt{m_k^y+2}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}m_k^y m_k^y\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}$$

$$+m_k^y\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y+1}\sqrt{m_k^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y+2}$$

$$+m_k^y\sqrt{m_k^y+1}\sqrt{m_k^y+2}m_k^y\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}$$

$$+m_k^y m_k^y\sqrt{m_k^y+1}\sqrt{m_k^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}+m_k^y m_k^y m_k^y\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y+1}\sqrt{m_k^y+2}(m_k^y+1)\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y}\sqrt{m_k^y-1}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y-2}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}m_k^y(m_k^y+1)\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}m_k^y\sqrt{m_k^y}\sqrt{m_k^y-1}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y-2}$$

$$+m_k^y\sqrt{m_k^y+1}\sqrt{m_k^y+2}(m_k^y+1)\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}$$

$$+m_k^y\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y}\sqrt{m_k^y-1}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y-2}$$

$$+m_k^y m_k^y(m_k^y+1)\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}$$

$$+m_k^y m_k^y\sqrt{m_k^y}\sqrt{m_k^y-1}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y-2}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}(m_k^y+1)\sqrt{m_k^y+1}\sqrt{m_k^y+2}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}(m_k^y+1)m_k^y\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y}\sqrt{m_k^y-1}\sqrt{m_k^y+1}\sqrt{m_k^y+2}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y-2}\hat{a}_{n_j^y,m_j^y+2}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y}\sqrt{m_k^y-1}m_k^y\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y-2}\hat{a}_{n_j^y,m_j^y}$$

$$+m_k^y(m_k^y+1)\sqrt{m_k^y+1}\sqrt{m_k^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}$$

$$+m_k^y(m_k^y+1)m_k^y\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}$$

$$+m_k^y\sqrt{m_k^y}\sqrt{m_k^y-1}\sqrt{m_k^y+1}\sqrt{m_k^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y-2}\hat{a}_{n_j^y,m_j^y+2}$$

$$+m_k^y\sqrt{m_k^y}\sqrt{m_k^y-1}m_k^y\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y-2}\hat{a}_{n_j^y,m_j^y}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}(m_k^y+1)(m_k^y+1)\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}(m_k^y+1)\sqrt{m_k^y}\sqrt{m_k^y-1}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y-2}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y}\sqrt{m_k^y-1}(m_k^y+1)\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y-2}\hat{a}_{n_j^y,m_j^y}$$

$$+\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y}\sqrt{m_k^y-1}\sqrt{m_k^y}\sqrt{m_k^y-1}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y-2}\hat{a}_{n_j^y,m_j^y-2}$$

$$+m_k^y(m_k^y+1)(m_k^y+1)\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}$$

$$+m_k^y(m_k^y+1)\sqrt{m_k^y}\sqrt{m_k^y-1}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y-2}$$

$$+m_k^y\sqrt{m_k^y}\sqrt{m_k^y-1}(m_k^y+1)\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y-2}\hat{a}_{n_j^y,m_j^y}$$

$$+m_k^y\sqrt{m_k^y}\sqrt{m_k^y-1}\sqrt{m_k^y}\sqrt{m_k^y-1}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y-2}\hat{a}_{n_j^y,m_j^y-2}$$

$$+(m_k^y+1)\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y+1}\sqrt{m_k^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y+2}$$

$$+(m_k^y+1)\sqrt{m_k^y+1}\sqrt{m_k^y+2}m_k^y\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}$$

$$+(m_k^y+1)m_k^y\sqrt{m_k^y+1}\sqrt{m_k^y+2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}$$

$$+(m_k^y+1)m_k^y m_k^y\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}$$

$$+\sqrt{m_k^y}\sqrt{m_k^y-1}\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y+1}\sqrt{m_k^y+2}\hat{a}_{n_j^y,m_j^y-2}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y+2}$$

$$+\sqrt{m_k^y}\sqrt{m_k^y-1}\sqrt{m_k^y+1}\sqrt{m_k^y+2}m_k^y\hat{a}_{n_j^y,m_j^y-2}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}$$

$$+\sqrt{m_k^y}\sqrt{m_k^y-1}m_k^y\sqrt{m_k^y+1}\sqrt{m_k^y+2}\hat{a}_{n_j^y,m_j^y-2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}$$

$$+\sqrt{m_k^y}\sqrt{m_k^y-1}m_k^y m_k^y\hat{a}_{n_j^y,m_j^y-2}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}$$

$$+(m_k^y+1)\sqrt{m_k^y+1}\sqrt{m_k^y+2}(m_k^y+1)\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y}$$

$$+(m_k^y+1)\sqrt{m_k^y+1}\sqrt{m_k^y+2}\sqrt{m_k^y}\sqrt{m_k^y-1}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y+2}\hat{a}_{n_j^y,m_j^y-2}$$

$$+(m_k^y+1)m_k^y(m_k^y+1)\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}\hat{a}_{n_j^y,m_j^y}$$

$$+(m_j+1)m_j\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}$$
$$+\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j+1}\sqrt{m_j+2}(m_j+1)\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j-1}$$
$$+\sqrt{m_j}\sqrt{m_j-1}m_j(m_j+1)\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j}\sqrt{m_j-1}m_j\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}$$
$$+(m_j+1)(m_j+1)\sqrt{m_j+1}\sqrt{m_j+2}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j+1}$$
$$+(m_j+1)(m_j+1)m_j\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$
$$+(m_j+1)\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j+1}\sqrt{m_j+2}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j+1}$$
$$+(m_j+1)\sqrt{m_j}\sqrt{m_j-1}m_j\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j}\sqrt{m_j-1}(m_j+1)\sqrt{m_j+1}\sqrt{m_j+2}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j+1}$$
$$+\sqrt{m_j}\sqrt{m_j-1}(m_j+1)m_j\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j+1}\sqrt{m_j+2}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j+1}$$
$$+\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j}\sqrt{m_j-1}m_j\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}$$
$$+(m_j+1)(m_j+1)(m_j+1)\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$
$$+(m_j+1)(m_j+1)\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}$$
$$+(m_j+1)\sqrt{m_j}\sqrt{m_j-1}(m_j+1)\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}$$
$$+(m_j+1)\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j-1}$$
$$+\sqrt{m_j}\sqrt{m_j-1}(m_j+1)(m_j+1)\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j}\sqrt{m_j-1}(m_j+1)\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}$$
$$+\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j}\sqrt{m_j-1}(m_j+1)\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j-1})\Big]$$

$$+(a_{22}+\eta_j f_{2})\sum_{j=1}^{3}\left[\frac{\hbar^3}{8\left(\frac{k_j}{m_j}\right)^{\frac{3}{2}}}\right.$$

$$\Big(\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j+1}\sqrt{m_j+2}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j+1}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j+1}\sqrt{m_j+2}m_j\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}m_j\sqrt{m_j+1}\sqrt{m_j+2}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j+1}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}m_jm_j\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$
$$+m_j\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j+1}\sqrt{m_j+2}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j+1}$$
$$+m_j\sqrt{m_j+1}\sqrt{m_j+2}m_j\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}$$
$$+m_jm_j\sqrt{m_j+1}\sqrt{m_j+2}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j+1}+m_jm_jm_j\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j+1}\sqrt{m_j+2}(m_j+1)\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j-1}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}m_j(m_j+1)\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}m_j\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}$$
$$+m_j\sqrt{m_j+1}\sqrt{m_j+2}(m_j+1)\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}$$
$$+m_j\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j-1}$$
$$+m_jm_j(m_j+1)\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$
$$+m_jm_j\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}(m_j+1)\sqrt{m_j+1}\sqrt{m_j+2}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j+1}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}(m_j+1)m_j\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j+1}\sqrt{m_j+2}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j+1}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j}\sqrt{m_j-1}m_j\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}$$
$$+m_j(m_j+1)\sqrt{m_j+1}\sqrt{m_j+2}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j+1}$$
$$+m_j(m_j+1)m_j\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$
$$+m_j\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j+1}\sqrt{m_j+2}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j+1}$$
$$+m_j\sqrt{m_j}\sqrt{m_j-1}m_j\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}(m_j+1)(m_j+1)\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}(m_j+1)\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j-1}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j}\sqrt{m_j-1}(m_j+1)\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j}$$
$$+\sqrt{m_j+1}\sqrt{m_j+2}\sqrt{m_j}\sqrt{m_j-1}\sqrt{m_j}\sqrt{m_j-1}\hat{a}_{n_j,m_j+1}\hat{a}_{n_j,m_j-1}\hat{a}_{n_j,m_j-1}$$
$$+m_j(m_j+1)(m_j+1)\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}\hat{a}_{n_j,m_j}$$

$$+m_i^a(m_i^a+1)\sqrt{m_i^a}\sqrt{m_i^a-1}\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a-2}$$
$$+m_i^a\sqrt{m_i^a}\sqrt{m_i^a-1}(m_i^a+1)\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}$$
$$+m_i^a\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a}\sqrt{m_i^a-1}\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a-2}$$
$$+(m_i^a+1)\sqrt{m_i^a+1}\sqrt{m_i^a+2}\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a+2}\delta_{n_i^a,m_i^a+2}$$
$$+(m_i^a+1)\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,m_i^a\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a+2}\delta_{n_i^a,m_i^a}$$
$$+(m_i^a+1)m_i^a\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a+2}$$
$$+(m_i^a+1)m_i^a m_i^a\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a+1}\sqrt{m_i^a+2}\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a+2}\delta_{n_i^a,m_i^a+2}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,m_i^a\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a+2}\delta_{n_i^a,m_i^a}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\,m_i^a\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a+2}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\,m_i^a m_i^a\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}$$
$$+(m_i^a+1)\sqrt{m_i^a+1}\sqrt{m_i^a+2}(m_i^a+1)\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a+2}\delta_{n_i^a,m_i^a}$$
$$+(m_i^a+1)\sqrt{m_i^a+1}\sqrt{m_i^a+2}\sqrt{m_i^a}\sqrt{m_i^a-1}\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a+2}\delta_{n_i^a,m_i^a-2}$$
$$+(m_i^a+1)m_i^a(m_i^a+1)\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}$$
$$+(m_i^a+1)m_i^a\sqrt{m_i^a}\sqrt{m_i^a-1}\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a-2}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a+1}\sqrt{m_i^a+2}(m_i^a+1)\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a+2}\delta_{n_i^a,m_i^a}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a+1}\sqrt{m_i^a+2}\sqrt{m_i^a}\sqrt{m_i^a-1}\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a+2}\delta_{n_i^a,m_i^a-2}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\,m_i^a(m_i^a+1)\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\,m_i^a\sqrt{m_i^a}\sqrt{m_i^a-1}\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a-2}$$
$$+(m_i^a+1)(m_i^a+1)\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a+2}$$
$$+(m_i^a+1)(m_i^a+1)m_i^a\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}$$
$$+(m_i^a+1)\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a+2}$$
$$+(m_i^a+1)\sqrt{m_i^a}\sqrt{m_i^a-1}\,m_i^a\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}(m_i^a+1)\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a+2}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}(m_i^a+1)m_i^a\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a+2}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a}\sqrt{m_i^a-1}\,m_i^a\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}$$
$$+(m_i^a+1)(m_i^a+1)(m_i^a+1)\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}$$
$$+(m_i^a+1)(m_i^a+1)\sqrt{m_i^a}\sqrt{m_i^a-1}\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a-2}$$
$$+(m_i^a+1)\sqrt{m_i^a}\sqrt{m_i^a-1}(m_i^a+1)\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}$$
$$+(m_i^a+1)\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a}\sqrt{m_i^a-1}\,\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a-2}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}(m_i^a+1)(m_i^a+1)\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}(m_i^a+1)\sqrt{m_i^a}\sqrt{m_i^a-1}\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}\delta_{n_i^a,m_i^a-2}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a}\sqrt{m_i^a-1}(m_i^a+1)\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a}\sqrt{m_i^a-1}\sqrt{m_i^a}\sqrt{m_i^a-1}\,\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a-2}\delta_{n_i^a,m_i^a-2})\Big] \qquad (15)$$

The equation (15) represents Hamiltonian operator matrix until sixth degree in case gauge theory (quarks and gluons plasma).

-To calculate the evolution of time for the average values for colored magnetic energy $\sum_{a=1}^{8}\sum_{i=1}^{3} B_i^a B_i^a$:

$$\sum_{a=1}^{8}\sum_{i=1}^{3} B_i B_i = \frac{\hbar}{2\sqrt{\frac{\omega}{\omega_i}}}\left(B_i^{+}B_i^{+} + B_i^{+}B_i + B_iB_i^{+} + B_iB_i\right)$$

- Depending on (11), (12) we find:

$$\langle n_i^a|B_iB_i|m_i^a\rangle = \frac{\hbar}{2\sqrt{\frac{\omega}{\omega_i}}}\Big[\Big(\sum_{a=1}^{8}\sum_{i=1}^{3}\sqrt{m_i^a+1}\sqrt{m_i^a+2}\,\delta_{n_i^a,m_i^a+2} + 2m_i^a\delta_{n_i^a,m_i^a}$$
$$+\sqrt{m_i^a}\sqrt{m_i^a-1}\,\delta_{n_i^a,m_i^a-2}\Big) + 0\Big] \qquad (16)$$

The equation (16) represents color magnetic energy matrix.

The colored magnetic energy matrix checks the equation:

$$i\hbar\frac{dB_iB_i}{dt} = (B_iB_i)H - H(B_iB_i) \qquad (17)$$

And thus:

$$i\hbar\frac{d}{dt}\langle \ldots n_i^a \ldots | B_i^a B_i^a | \ldots m_i^a \ldots \rangle = \langle \ldots n_i^a \ldots | (B_i^a B_i^a) H | \ldots m_i^a \ldots \rangle - \langle \ldots n_i^a \ldots | H (B_i^a B_i^a) | \ldots m_i^a \ldots \rangle$$

$$= \sum_{n_i'^a} [\langle \ldots n_i^a \ldots | B_i^a B_i^a | \ldots n_i'^a \ldots \rangle \langle \ldots n_i'^a \ldots | H | \ldots m_i^a \ldots \rangle -$$

$$\langle \ldots n_i^a \ldots | H | \ldots n_i'^a \ldots \rangle \langle \ldots n_i'^a \ldots | B_i^a B_i^a | \ldots m_i^a \ldots \rangle]$$

$$\frac{dB_i^a B_i^a}{dt} = -\frac{i}{\hbar}\sum_{n_i'^a} \left( (B_i^a B_i^a)_{n_i^a n_i'^a} H_{n_i'^a m_i^a} - H_{n_i^a n_i'^a} (B_i^a B_i^a)_{n_i'^a m_i^a} \right) \qquad (18)$$

We can calculate the evolution of time for the average values in equation (18) numerically by MATLAB program after calculating the numerical values of matrix (15),(16) by Fortran(77) language program.

We have got [10]: $1GeV^{-1}=6.58x10^{-25}$ sec

We draw graphs which represent the evolution of time of magnetic energy BB for levels n=0,1,2 for different values for coupling constant g we find at what time the phase change occurs and thus we get the life time of the quarks and gluons plasma:

We conclude from these forms that for small and large sizes for L=2.656 fm which corresponds them g=0.513652004 to L=1.86 fm which corresponds them g=2.597495692 is being life of quarks and gluons plasma of rank $10^{-29}$sec, Because phase change occurs at this time after this time we move on to phase of the Hadrons.

## 4 CONCLUSIONS AND RECOMMENDATIONS:

This study is the first in quantum mechanics which takes a numerical study of the evolution of the real time for gauge theory with group SU(2) i.e. nine freedom degrees, we recommend a numerical study of the evolution of the real time for gauge theory with group SU(3) i.e. twenty four freedom degree.

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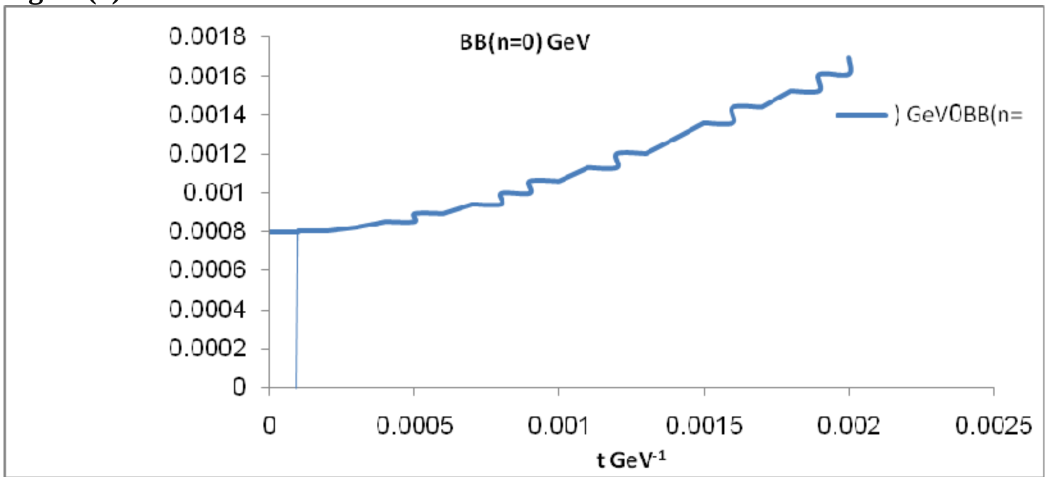
**Figure legends:**

**Figure(1):** shows the evolution of time for the average values of colored magnetic energy for ground level dependency the time for g=0.649301461.

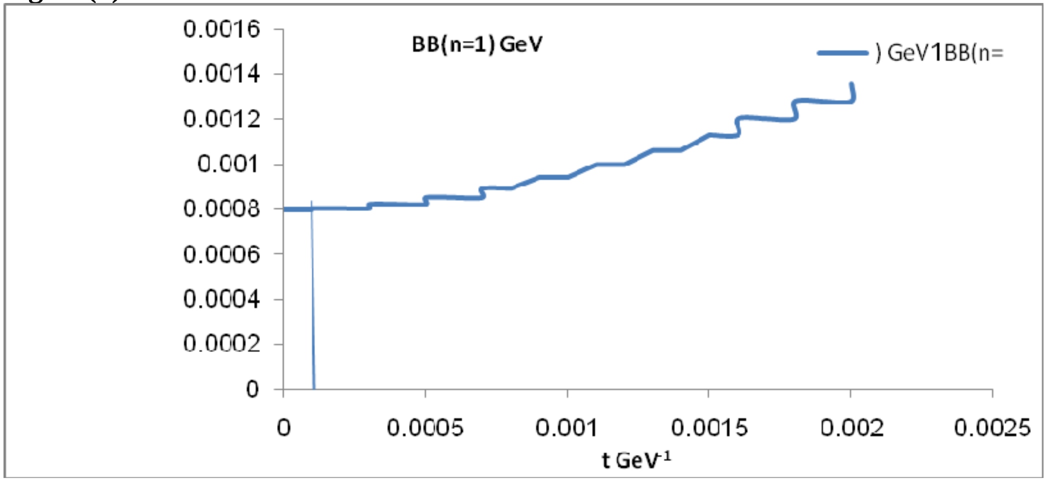
**Figure(2):** shows the evolution of time for the average values of colored magnetic energy for first level dependency the time for g=0.649301461.

**Figure(3):** shows the evolution of time for the average values of colored magnetic energy for second level dependency the time for g=0.649301461.

**Figure(1)**



**Figure(2)**




**Figure(3)**